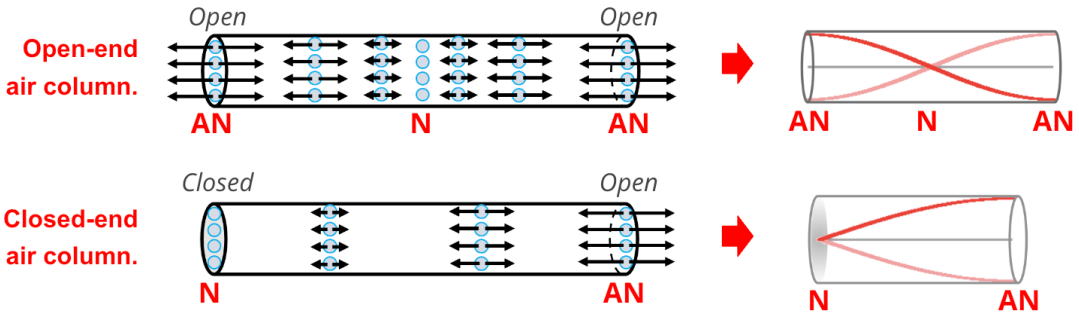
Closed-End Air Columns
Lesson Notes

Learning Outcomes

- How do you draw the standing wave patterns for the various harmonics of a closed-end air column?
- How are the frequencies and wavelengths for the various harmonics related?

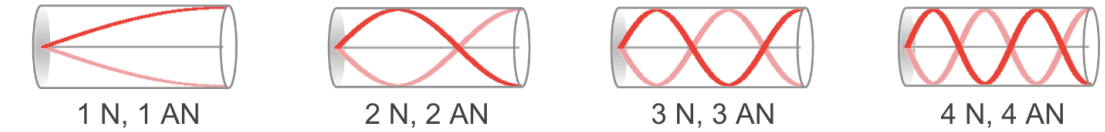
Open-End vs. Closed-End Air Columns

- Open-End Air Columns: both ends are open to the surrounding air.
- Closed-End Air Columns: one end is open and one end is closed.
- Open ends are **anti-nodes** (**AN**); air vibrates in and out of the air column.
- Closed ends are **nodes** (**N**); air does not vibrate at the closed end.



Standing Wave Patterns for Closed-End Air Columns

There are several natural frequencies or **harmonics** that will force the closed-end air column to resonate as a standing wave:



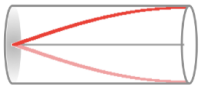
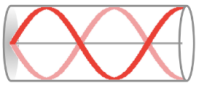
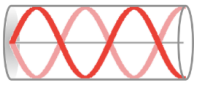
Fundamental (1st H)	3rd Harmonic	5th Harmonic	7th Harmonic
1 N, 1 AN	2 N, 2 AN	3 N, 3 AN	4 N, 4 AN
$L = 0.25 \cdot \lambda_1$	$L = 0.75 \cdot \lambda_3$	$L = 1.25 \cdot \lambda_5$	$L = 1.75 \cdot \lambda_7$
↓ algebra	↓ algebra	↓ algebra	↓ algebra
$\lambda_1 = (4/1) \cdot L$	$\lambda_3 = (4/3) \cdot L$	$\lambda_5 = (4/5) \cdot L$	$\lambda_7 = (4/7) \cdot L$

If the air column is 1.2 m long (L=1.2 m), then …

$\lambda_1 = 4.80$ m	$\lambda_3 = 1.60$ m	$\lambda_5 = 0.96$ m	$\lambda_7 = 0.69$ m

Frequency Relationships for Closed-End Air Columns

There are very clear wavelength and frequency relationships between the harmonics of an closed-end air column.

Fundamental (1st H)	3rd Harmonic	5th Harmonic	7th Harmonic
			
$\lambda_1 = (4/1)\bullet L$	$\lambda_3 = (4/3)\bullet L$	$\lambda_5 = (4/5)\bullet L$	$\lambda_7 = (4/7)\bullet L$

3rd Harmonic: one-third the **λ** of 1st Harmonic … and three times the **f**.

5th Harmonic: one-fifth the **λ** of 1st Harmonic … and five times the **f**.

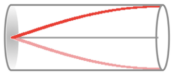
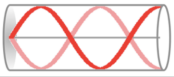
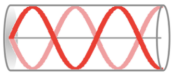
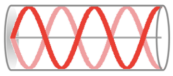
7th Harmonic: one-seventh the **λ** of 1st Harmonic … and seven times the **f**.

If the fundamental frequency is 200 Hz, then …

$f_1 = 200$ Hz $f_3 = 600$ Hz $f_5 = 1000$ Hz $f_7 = 1400$ Hz

Mathematics of Closed-End Air Columns

Two general equations for **nth** harmonic: $\lambda_n = (4/n)\bullet L$ $f_n = n\bullet f_1$

Harmonic	Pattern	# of Nodes	# of Antinodes	λ	f	Examples	
						λ (m)	f (Hz)
1st		1	1	λ_1	f_1	2.40	150
3rd		2	2	$\lambda_1/3$	$3\bullet f_1$	0.80	300
5th		3	3	$\lambda_1/5$	$5\bullet f_1$	0.48	450
7th		4	4	$\lambda_1/7$	$7\bullet f_1$	0.343	600
9th		5	5	$\lambda_1/9$	$9\bullet f_1$	0.267	750
nth	--	(n+1)/2	(n+1)/2	λ_1/n	$n\bullet f_1$	2.40/n	150•n

Based on L = 60 cm

Assume $f_1 = 150$ Hz