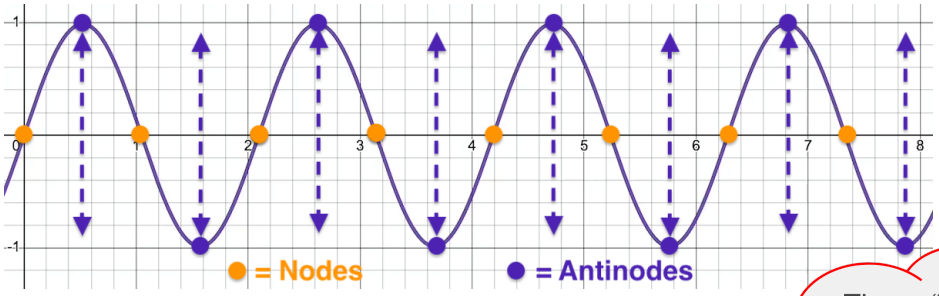
Harmonic Frequencies and Standing Waves
Lesson Notes

Learning Outcomes
- What is a harmonic and how are they related to standing waves?
- What are the mathematical patterns derived from standing wave patterns?

Standing Wave Formation
When a traveling wave is confined to a small space, a regular and repeating pattern can be observed in the medium if vibrated *at just the right frequency*.



Standing Wave
A wave pattern with permanently-positioned points that appear to be standing still.

Standing waves consist of …
- **Nodes** (points of placement)
- **Antinodes** (points with maximum +/- displacement)

These "just the right frequencies" are the **natural frequencies** of the string. We refer to them as **harmonics**.

Chladni Plates Demonstration
A metal plate is firmly mounted to a lab bench, covered with salt, and strummed with a violin bow.

The plate vibrates with one of its natural or harmonic frequencies and the salt forms a pattern.

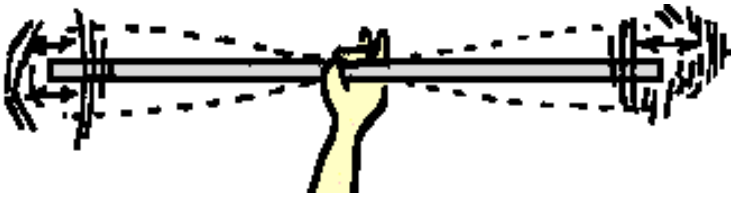
The procedure is repeated and a new pattern is observed.

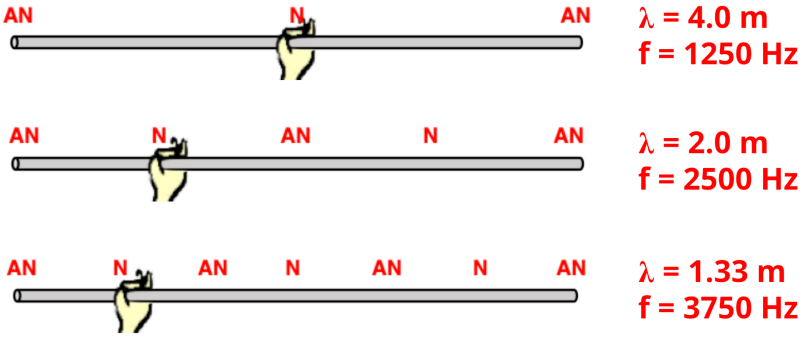
Each harmonic frequency is associate with a standing wave pattern with nodes (salt lines) and antinodes.

Each pattern represents the **lowest energy, favored mode of vibration** of the metal plate.

Singing Aluminum Pole Demonstration

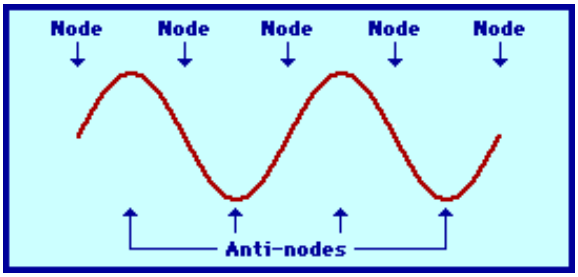
- A 2-m long, hollow aluminum pole is held at its midpoint and *strummed* to set it into vibrational motion.
- The procedure is repeated with the pole held at different positions (¼-th, ⅙-th, etc.). Different pitches are heard for the different positions where it is held.
- Each vibrational pattern represents a low-energy, favored vibrational modes.
- Each pattern is associated with a harmonic frequency.





Vibrating Strings

The natural frequencies or harmonics of a vibrating string are associated with standing wave patterns.



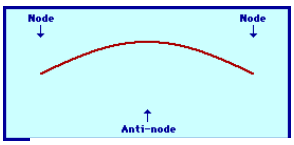
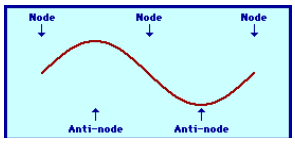
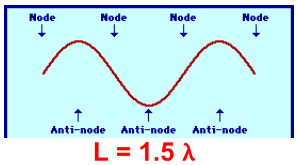
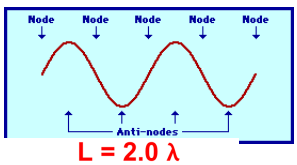
One of the several *favored* vibrational modes is shown at the right.

All patterns share these traits:

- There is an alternating pattern of nodes and antinodes.
- A half-number or a whole number of wavelengths.
- Nodal positions are at the ends of the string.
- One pattern is related to the next pattern by the addition (or subtraction) of one or more nodes (and antinodes).

Harmonics for Vibrating Strings

There are many other vibrational patterns with which the string can vibrate:

1st Harmonic	2nd Harmonic	3rd Harmnoic	4th Harmonic
			
L = 0.5 λ	L = 1.0 λ	L = 1.5 λ	L = 2.0 λ
algebra	*algebra*	*algebra*	*algebra*
λ = 2.0•L	λ = L	λ = (⅔)•L	λ = (½)•L

2nd Harmonic: one-half the **λ** of 1st Harmonic … and two times the **f**.

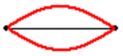
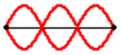
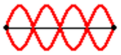
3rd Harmonic: one-third the **λ** of 1st Harmonic … and three times the **f**.

4th Harmonic: one-fourth the **λ** of 1st Harmonic … and four times the **f**.

Mathematical Patterns

The manner in which the string vibrates exhibits a pattern …
… and so does the mathematics associated with n, f, and λ.

String Length = 60 cm = 0.60 m

Harmonic	Pattern	# of Nodes	# of Antinodes	λ	f	Examples	
						λ (m)	f (Hz)
1st		2	1	λ_1	f_1	1.20	50
2nd		3	2	$\lambda_1/2$	$2 \cdot f_1$	0.60	100
3rd		4	3	$\lambda_1/3$	$3 \cdot f_1$	0.40	150
4th		5	4	$\lambda_1/4$	$4 \cdot f_1$	0.30	200
nth	**--**	**n+1**	**n**	**λ_1/n**	**$n \cdot f_1$**	**1.20/n**	**50·n**