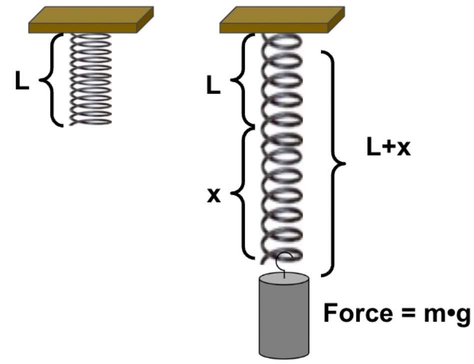
Vibrating Mass on a Spring
Lesson Notes

Learning Outcomes

- How does the force, acceleration, speed, position, and kinetic and potential energy of a vibrating mass on a spring change over the course of a cycle?

Hooke's Law

- In the absence of a force, a spring assumes a natural length.
- When a force is applied (e.g., by adding a mass), the spring stretches (or compresses) by a distance of x.
- The spring exerts a force in the opposite direction as the direction of its stretch (or compression).
- Hooke's Law states the relationship between the amount of stretch and the amount of force applied by the spring.



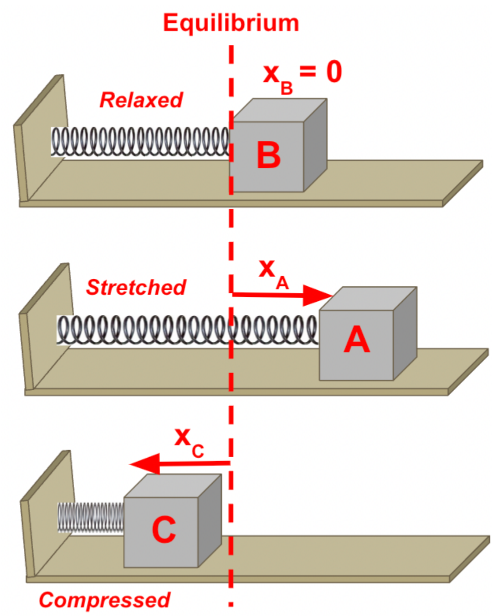
$F_{spring} = -k \cdot x$

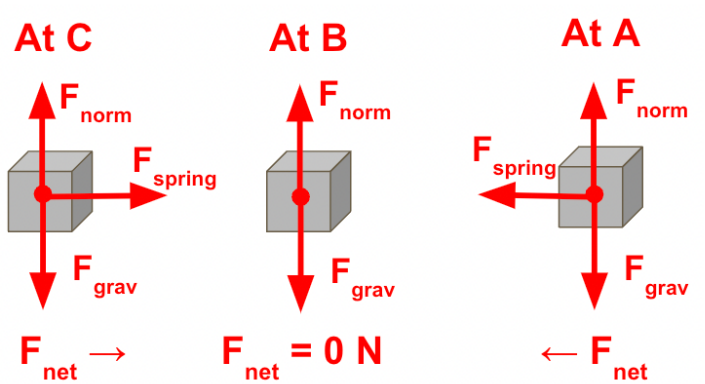
k = spring constant (N/m)

x = displacement

Force Analysis – Horizontal Springs

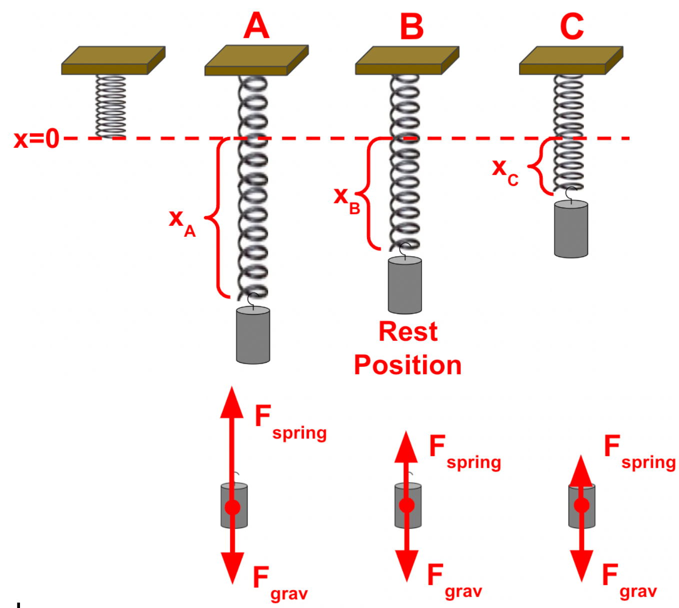
- Consider a mass attached to a **relaxed** spring on a friction-free air table.
- If the spring is stretched and then released, it will begin vibrating back and forth between its two extremes.





Force Analysis – Vertical Springs

- A spring is suspended vertically and assumes its unstretched length.
- A mass is hung on the spring and lowered to a **rest position**. The spring is stretched to position B.
- The mass is pulled to A and released from rest. It oscillates back and forth between positions A and C.

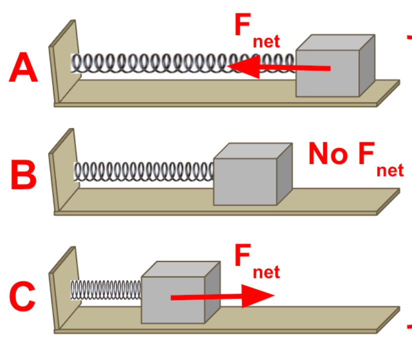


F_{grav}: always down F_{spring}: always up

F_{net}: always towards B (restoring force)

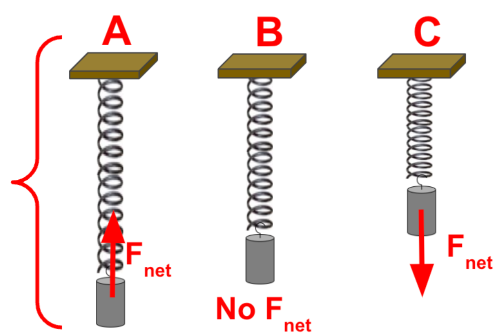
Speed and Acceleration

As the mass vibrates back and forth, its speed changes.
The speed is 0 m/s at the extreme positions and a maximum value at the equilibrium position.



A to B and C to B: mass speeds up.

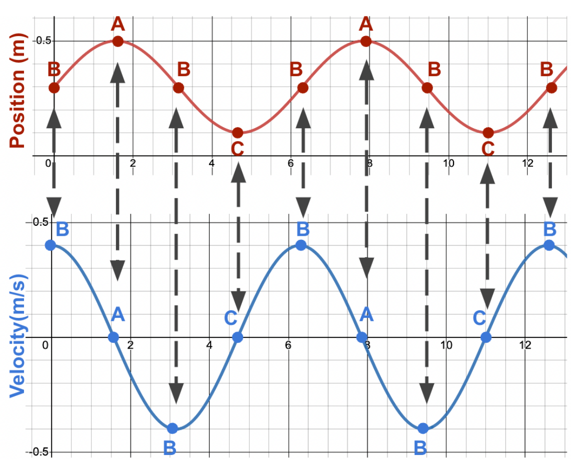
B to A and B to C: mass slows down.

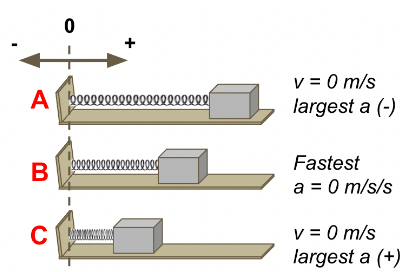


The acceleration is in the direction of and proportional to the net force (restoring force). Like F_{net}, acceleration is always directed towards the equilibrium position. And like F_{net}, acceleration is largest at the extremes and 0 m/s/s at the equilibrium position.

Sinusoidal Nature of a Mass on a Spring

Position and velocity (and more) change periodically as a function of the sine of time.





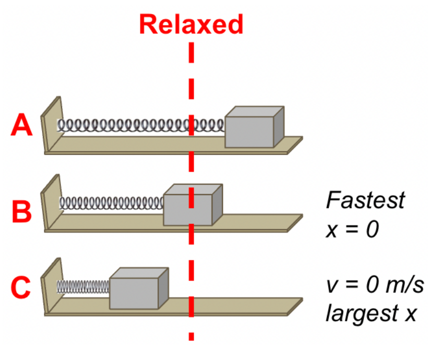
A to B and C to B: mass speeds up.

B to A and B to C: mass slows down.

Energy Analysis (Horizontal Springs)

As the mass vibrates back and forth between extremes, energy is changing from **elastic potential energy** (**PE_{spring}**) and **kinetic energy** (**KE**).

- **Kinetic energy** (speed dependent) is greatest at position B.
- **Elastic potential energy** (stretch/compression dependent) is greatest at positions A and C.



A to B: PE_{spring}↓ and KE↑

B to C: PE_{spring}↑ and KE↓

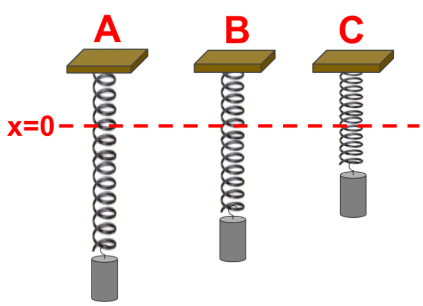
C to B: PE_{spring} ↓ and KE ↑

B to A: PE_{spring}↑ and KE↓

Energy Analysis (Vertical Springs)

As a mass on a vertical spring vibrates between extremes, energy is changing between **gravitational potential energy** (**PE_{grav}**), **elastic potential energy** (**PE_{spring}**) and **kinetic energy** (**KE**).

- **Kinetic energy** (speed dependent) is greatest at position B.
- **Gravitational potential energy** (height dependent) is greatest at positions C.
- **Elastic potential energy** (stretch/compression dependent) is greatest at position A and least at position C.



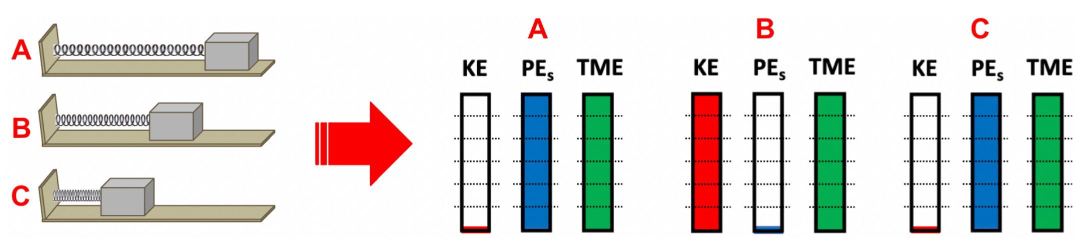
A to B: PE_{spring}↓ , PE_{grav}↑ , and KE ↑

B to C: PE_{spring}↓ , PE_{grav}↑ , and KE ↓

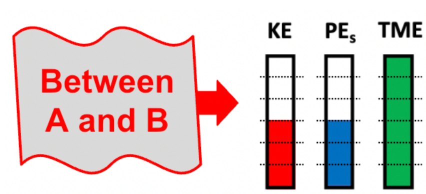
C to B: PE_{spring}↑, PE_{grav} ↓ , and KE ↑

B to A: PE_{spring}↑, PE_{grav} ↓ , and KE ↓

Energy Bar Charts



As the **kinetic energy** (**KE**) increases, the **elastic potential energy** (**PE_{spring}**) decreases, but the **total mechanical energy** (**TME**) remains constant. And vice versa.



Period and Frequency

The period (**T**) of vibration of a spring-mass system depends upon the spring constant (**k**) of the spring and the mass (**m**) of the vibrating object.

$$T = 2 \cdot \pi \cdot \sqrt{(m/k)}$$

More massive objects will have longer periods.
Stiffer springs (larger k) will have shorter periods.

The frequency (**f**) of vibration is the reciprocal of the period and calculated as …

$$f = 1/T = \sqrt{(k/m)} \; / \; 2 \cdot \pi$$