

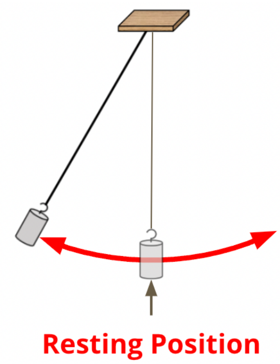
Pendulum Motion
Lesson Notes

Learning Outcomes

- How does the force, acceleration, position, velocity, kinetic energy, and potential energy change over the course of a pendulum's path?
- What factors affect the period of a pendulum and how?

The Simple Pendulum

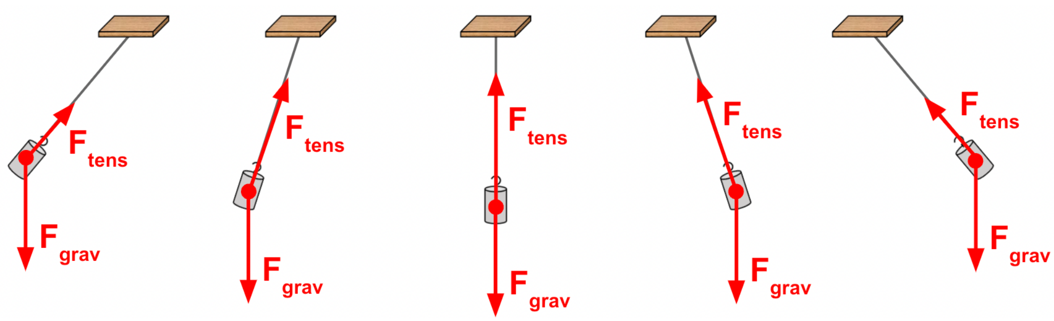
- A simple pendulum consists of a relatively massive object (known as the **bob**) suspended from a light string that is secured to a support.
- The bob swings back and forth along a circular arc about a fixed position.
- The fixed position is the **resting position**; it is the position of the bob when the string is vertical.
- The pendulum's motion is an example of **periodic motion** - **repeating** (occurring over and over again) and **regular** (the period of each cycle is the same).



Free-Body Diagrams

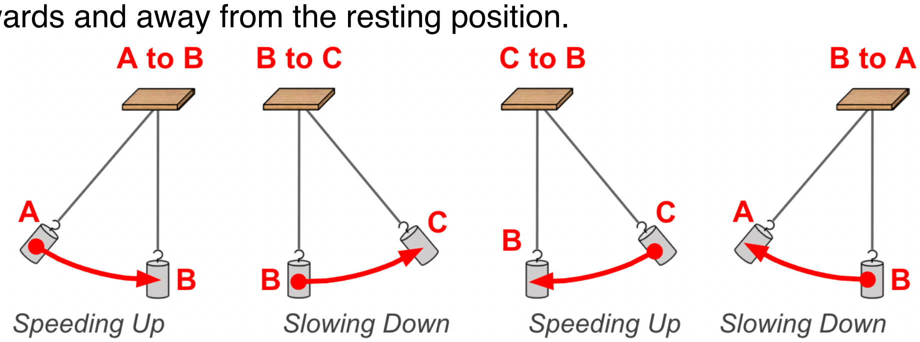
Two dominant forces act upon the bob - the force of gravity (**F_{grav}**) and tension (**F_{tens}**).

- **F_{grav}**: constant magnitude (m•9.8 N/kg); always directed downward.
- **F_{tens}**: variable magnitude; always directed upwards towards pivot point.



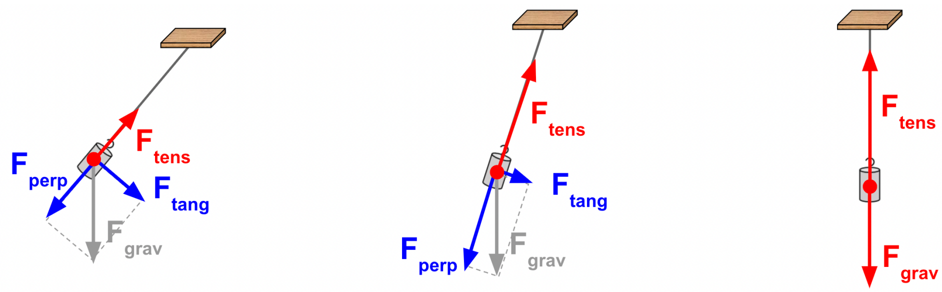
Acceleration and Net Force

- A net force (**F_{net}**) causes an acceleration (**a**) and an acceleration requires a net force.
- The direction of the **F_{net}** and **a** vectors are the same.
- There is a centripetal **a** and **F_{net}** since the object is moving along a circular path.
- There is a tangential **a** and **F_{net}** since the object is speeding up and slowing down as it moves towards and away from the resting position.



Force Analysis

A ⊥ axes system is drawn that is directed **tangential** to the circular arc and towards the pivot point (**perpendicular**). F_{grav} is resolved into two components along these axes.



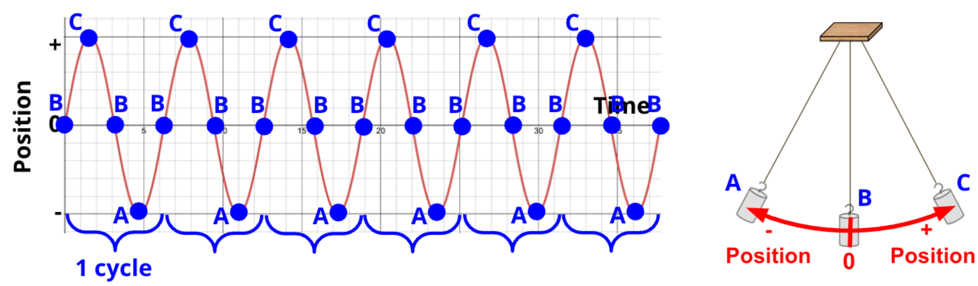
v = 0 … $F_{perp} = F_{tens}$ — F_{tang} is the restoring force

v ≠ 0 … $F_{tens} > F_{perp}$ — F_{tang} is the restoring force

v ≠ 0 … $F_{tens} > F_{perp}$ — No restoring force

Position vs. Time

A pendulum bob's position varies with time in a sinusoidal manner.



B to **C**: moving from rest to extreme right ➡ **Slowing down**

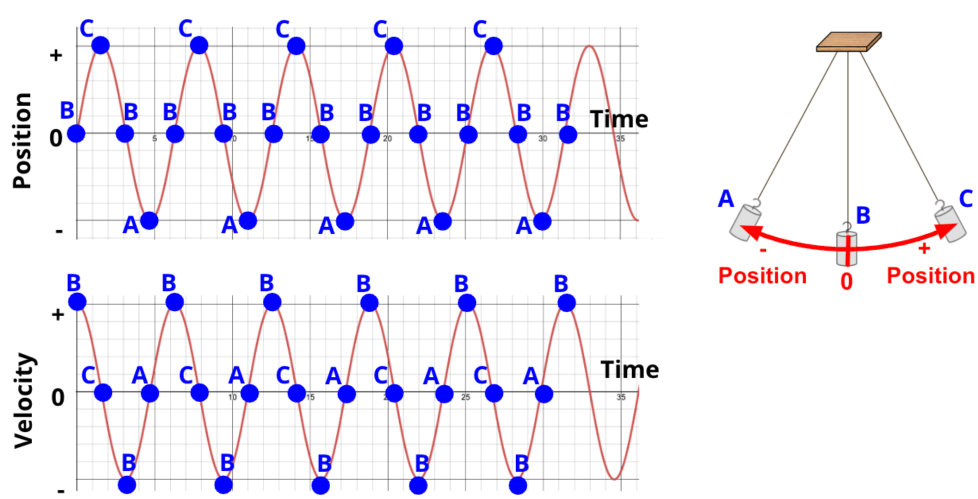
C to **B**: moving from extreme right to rest ➡ **Speeding up**

B to **A**: moving from rest to extreme left ➡ **Slowing down**

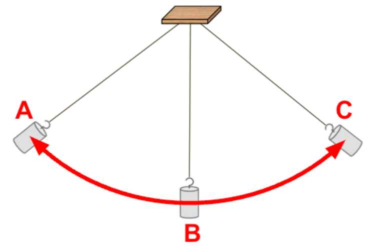
A to **B**: moving from extreme left to rest ➡ **Speeding up**

Velocity vs. Time

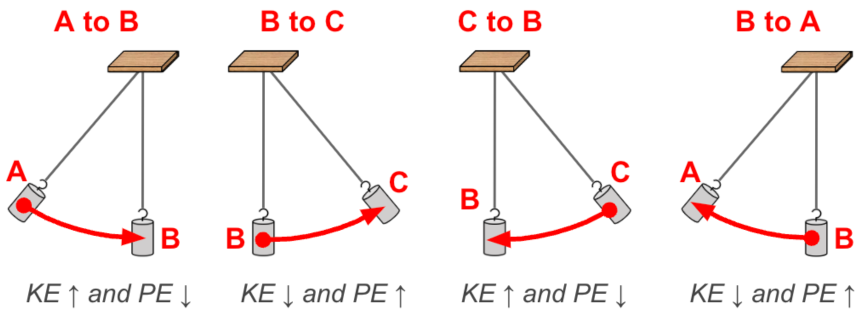
A v-t graph becomes sensible when you comprehend the p-t graph.



Energy Analysis of a Pendulum

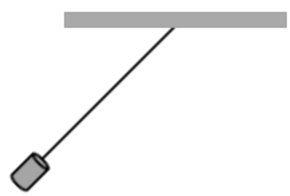


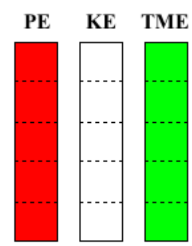
- If we ignore damping effects, the only forces doing work on a pendulum is gravity.
- Being a conservative force, F_{grav} does not remove mechanical energy from the system.



Energy Bar Charts

Energy bar charts are conceptual tools used to convey the energy stores of a system and how they change over time.





Note the following patterns:

- As the bob moves towards rest position: PE↓, KE↑.
- As the bob moves away from rest position: PE↑, KE↓.
- While KE and PE change, the sum of the two forms remain constant.

  (The above neglects damping effects.)

Period of a Pendulum

- **Period (T)**: The time to complete a full cycle of vibration.
- The pendulum swings back and forth with the same period cycle after cycle.
- A common Physics lab involves investigating the effect of string length, bob mass, and swing angle upon the period.
- A pendulum's period (**T**) depends upon the string length (**L**). In fact, $T \propto \sqrt{L}$.
  - Doubling **L** causes **T** to increase by $\sqrt{2}$.
  - Tripling **L** causes **T** to increase by $\sqrt{3}$.
  - Quadrupling **L** causes **T** to increase by $\sqrt{4}$.
  - Halving **L** causes **T** to decrease by $\sqrt{2}$ (i.e. $1/\sqrt{2}$ of the original value).

The formula relating T to L is: $T = 2 \cdot \pi \sqrt{(L/g)}$ where g =9.8 m/s/s.