Kepler's Law of Harmonies

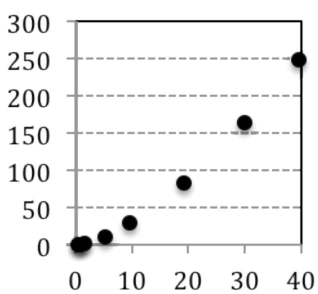
In the late 1600s, German mathematician Johannes Kepler acquired the carefully collected planetary data of Danish astronomer Tycho Brahe. Kepler attempted to analyze the known planetary information of that day. One of his challenges was to identify what appeared to be a mathematical relationship between the **period** (time for a complete orbit) of each planet and its average (or *mean*) distance from the sun. **Table 1** shows a *more polished* form of the data that resulted from his analysis.

Table 1

Planet	Mean Distance from Sun (au)	Period (year)
Mercury	0.39	0.241
Venus	0.72	0.615
Earth	1	1
Mars	1.52	1.88
Jupiter	5.2	11.8
Saturn	9.54	29.5
Uranus	19.18	84
Neptune	30.06	165
Pluto	39.44	248

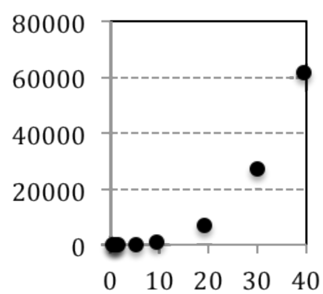
Some physics students were assigned the task of plotting the values of period (**T**) and distance (**R**) raised to various powers. Their goal was to find a linear relationship between the two quantities so that they could use the slope-intercept equation to relate them mathematically. They performed a linear regression analysis for each plot; the slope (**m**) and correlation coefficient (**COR**) are reported.

Figure 1
T vs. R



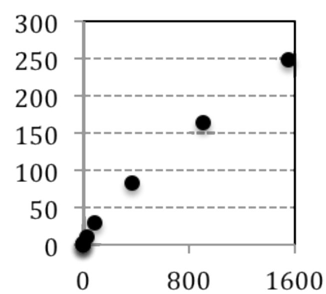
m = 6.12
COR = 0.9777

Figure 2
T^2 vs. R



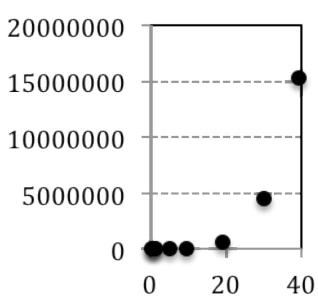
m = 1.34E+3
COR = 0.845

Figure 3
T vs. R^2



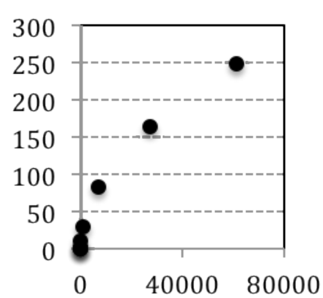
m = 0.162
COR = 0.988

Figure 4
T^3 vs. R



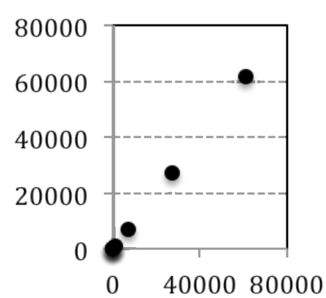
m = 3.02E+5
COR = 0.738

Figure 5
T vs. R^3



m = 4.12E-3
COR = 0.932

Figure 6
T^2 vs. R^3



m = 1.00
COR = 1.000

Questions:

1. Based on values given in the table, what do you suppose the definition is of an **astronomical unit (au)**?
   a. An au is defined as a very large distance.
   b. An au is defined as the distance to the edge of our solar system.
   c. An au is defined as the average distance between Earth and Sun.
   d. An au is defined as the average distance that any planet is from the Sun.

2. What affect does the mean distance that a planet is from the Sun seem to have upon its period of orbit?
   a. The period is greatest when the mean distance from the Sun is greatest.
   b. The period is smallest when the mean distance from the Sun is greatest.
   c. The period is definitely not affected by the mean distance from the Sun.
   d. There is no clear and obvious manner in which these variables are related.

3. Which plot shows the best fit for linear regression and why?
   a. The plot of **Figure 3** since it is the only plot that has a convex shape.
   b. The plot of **Figure 4** since it is the one with the greatest slope.
   c. The plot of **Figure 6** since the plotted points fit a straight line as noted by the COR.
   d. The plots of **Figure 2** and **Figure 4** since these are the plots that have the most dramatic curve and the largest slopes.

4. What importance does the correlation coefficient have to this study?
   a. It is an indication of the value of the y-coordinate when the x-coordinate is 0.
   b. It is an indication of how much error there is; a value that is close to 0 is best.
   c. It is an indication of the validity of the data; a value that is close to the slope value is best.
   d. It is an indication of how well the plotted points fit the regression line; a value close to 1.0 is best.

5. Once students completed the task of analyzing the data for the planets' orbit about the Sun, they were given data for the moons of Jupiter orbiting Jupiter. When they constructed plots similar to those in **Figure 1** through **Figure 6**, they observed that the plots had an identical shape as those in **Figure 1** through **Figure 6**. What conclusion could be drawn from such an observation?
   a. They probably forgot to clear the original data from the graphing program.
   b. There must have been an error in the collection of the data.
   c. The laws that govern the orbit of the moons must be identical to those that govern the planets' motion.
   d. If one analyzes enough data, they will always run across a numerical coincidence.

Table 2

Planet	Mean Distance from Star (au)	Period (year)
A	0.50	0.544
B	1.00	1.00
C	1.50	1.84
D	2.00	2.83
E	3.00	5.20
F	4.00	8.00
G	6.00	14.7

In order to demonstrate the meaningfulness of the data in **Table 1**, a physics teacher presented the data in **Table 2** at the right. The data are simpler and unique to a fictional solar system. Yet the data convey the same meaning as the data in **Table 1**. Use **Table 2** to answer the next several questions. A calculator may prove useful on some of the questions.

6. Which of the following two planets in **Table 2** have the same radius ratio as the radius ratio of Planet A and Planet C?
   a. Planet A and Planet E
   b. Planet C and Planet F
   c. Planet D and Planet G
   d. Planet E and Planet G

7. Which of the following two planets in **Table 1** have a radius ratio that is most similar to the radius ratio of Planet B and Planet F?
   a. Mercury and Earth
   b. Mars and Saturn
   c. Mars and Pluto
   d. Saturn and Pluto

8. Which of the following two planets in **Table 1** have a radius ratio that is most similar to the radius ratio of Planet D and Planet E?
   a. Mercury and Mars
   b. Earth and Jupiter
   c. Saturn and Pluto
   d. Uranus and Neptune

9. If the radius of a planet (such as one in **Table 2**) were somehow tripled, then the period of that planet would increase by a factor of _____.
   a. 1.5
   b. 3.0
   c. 5.2
   d. 9.0

10. What is the ratio of the periods of two planets that have a radius ratio of 3:1?
   a. The period ratio will be 1.84:1.
   b. The period ratio will be 3:1.
   c. The period ratio will be 5.20:1.
   d. The period ratio will be 9:1.

11. Which one of the following equations describing the relationship between the radius and the period of two planets (1 and 2) is most consistent with the data in **Table 2** and **Figure 1** to **Figure 6**?
   a. $\frac{R_1^2}{R_2^2}=\frac{T_1^2}{T_2^2}$
   b. $\frac{R_1^3}{R_2^3}=\frac{T_1^2}{T_2^2}$
   c. $\frac{R_1^2}{R_2^2}=\frac{T_2^3}{T_1^3}$
   d. $\frac{R_1^2}{R_2^2}=\frac{T_1^3}{T_2^3}$